



ÉCOLE POLYTECHNIQUE
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Elements of Programming

Principles of Functional Programming

Elements of Programming

Every non-trivial programming language provides:

- ▶ primitive expressions representing the simplest elements
- ▶ ways to *combine* expressions
- ▶ ways to *abstract* expressions, which introduce a name for an expression by which it can then be referred to.

The Read-Eval-Print Loop

Functional programming is a bit like using a calculator

An interactive shell (or REPL, for Read-Eval-Print-Loop) lets one write expressions and responds with their value.

The Scala REPL can be started by simply typing

```
> scala
```

Expressions

Here are some simple interactions with the REPL

```
scala> 87 + 145  
232
```

Functional programming languages are more than simple calculators because they let one define values and functions:

```
scala> def size = 2  
size: => Int
```

```
scala> 5 * size  
10
```

Evaluation

A non-primitive expression is evaluated as follows.

1. Take the leftmost operator
2. Evaluate its operands (left before right)
3. Apply the operator to the operands

A name is evaluated by replacing it with the right hand side of its definition

The evaluation process stops once it results in a value

A value is a number (for the moment)

Later on we will consider also other kinds of values

Example

Here is the evaluation of an arithmetic expression:

`(2 * pi) * radius`

Example

Here is the evaluation of an arithmetic expression:

$(2 * \text{pi}) * \text{radius}$

$(2 * 3.14159) * \text{radius}$

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Example

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$(2 * \pi) * \text{radius}$

$(2 * 3.14159) * \text{radius}$

$6.28318 * \text{radius}$

$6.28318 * 10$

62.8318

Parameters

Definitions can have parameters. For instance:

```
scala> def square(x: Double) = x * x  
square: (Double)Double
```

```
scala> square(2)  
4.0
```

```
scala> square(5 + 4)  
81.0
```

```
scala> square(square(4))  
256.0
```

```
scala> def sumOfSquares(x: Double, y: Double) = square(x) + square(y)  
sumOfSquares: (Double, Double)Double
```

Parameter and Return Types

Function parameters come with their type, which is given after a colon

```
def power(x: Double, y: Int): Double = ...
```

If a return type is given, it follows the parameter list.

Primitive types are as in Java, but are written capitalized, e.g:

Int	32-bit integers
Double	64-bit floating point numbers
Boolean	boolean values true and false

Evaluation of Function Applications

Applications of parameterized functions are evaluated in a similar way as operators:

1. Evaluate all function arguments, from left to right
2. Replace the function application by the function's right-hand side, and, at the same time
3. Replace the formal parameters of the function by the actual arguments.

Example

```
sumOfSquares(3, 2+2)
```

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

`square(3) + square(4)`

Example

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`3 * 3 + square(4)`

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

`square(3) + square(4)`

`3 * 3 + square(4)`

`9 + square(4)`

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

`square(3) + square(4)`

`3 * 3 + square(4)`

`9 + square(4)`

`9 + 4 * 4`

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

`square(3) + square(4)`

`3 * 3 + square(4)`

`9 + square(4)`

`9 + 4 * 4`

`9 + 16`

Example

`sumOfSquares(3, 2+2)`

`sumOfSquares(3, 4)`

`square(3) + square(4)`

`3 * 3 + square(4)`

`9 + square(4)`

`9 + 4 * 4`

`9 + 16`

`25`

The substitution model

This scheme of expression evaluation is called the *substitution model*.

The idea underlying this model is that all evaluation does is *reduce an expression to a value*.

It can be applied to all expressions, as long as they have no side effects.

The substitution model is formalized in the *λ -calculus*, which gives a foundation for functional programming.

Termination

- ▶ *Does every expression reduce to a value (in a finite number of steps)?*

Termination

- ▶ *Does every expression reduce to a value (in a finite number of steps)?*
- ▶ *No. Here is a counter-example*

```
def loop: Int = loop
```

```
loop
```


Changing the evaluation strategy

The interpreter reduces function arguments to values before rewriting the function application.

One could alternatively apply the function to unreduced arguments.

For instance:

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For instance:

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9 + (2+2) * (2+2)
```

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9 + 4 * (2+2)
```

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For instance:

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sumOfSquares(3, 2+2)
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```

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For instance:

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sumOfSquares(3, 2+2)
square(3) + square(2+2)
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
9 + 4 * (2+2)
9 + 4 * 4
25
```


Call-by-name and call-by-value

The first evaluation strategy is known as *call-by-value*, the second is known as *call-by-name*.

Both strategies reduce to the same final values as long as

- ▶ the reduced expression consists of pure functions, and
- ▶ both evaluations terminate.

Call-by-value has the advantage that it evaluates every function argument only once.

Call-by-name has the advantage that a function argument is not evaluated if the corresponding parameter is unused in the evaluation of the function body.

Call-by-name vs call-by-value

Question: Say you are given the following function definition:

```
def test(x: Int, y: Int) = x * x
```

For each of the following function applications, indicate which evaluation strategy is fastest (has the fewest reduction steps)

CBV	CBN	same	
fastest	fastest	#steps	
0	0	0	test(2, 3)
0	0	0	test(3+4, 8)
0	0	0	test(7, 2*4)
0	0	0	test(3+4, 2*4)

Call-by-name vs call-by-value

```
def test(x: Int, y: Int) = x * x
```

```
test(2, 3)
```

```
test(3+4, 8)
```

```
test(7, 2*4)
```

```
test(3+4, 2*4)
```