Computer Language Processing

Exercise Sheet 02 - Solutions

October 6, 2022

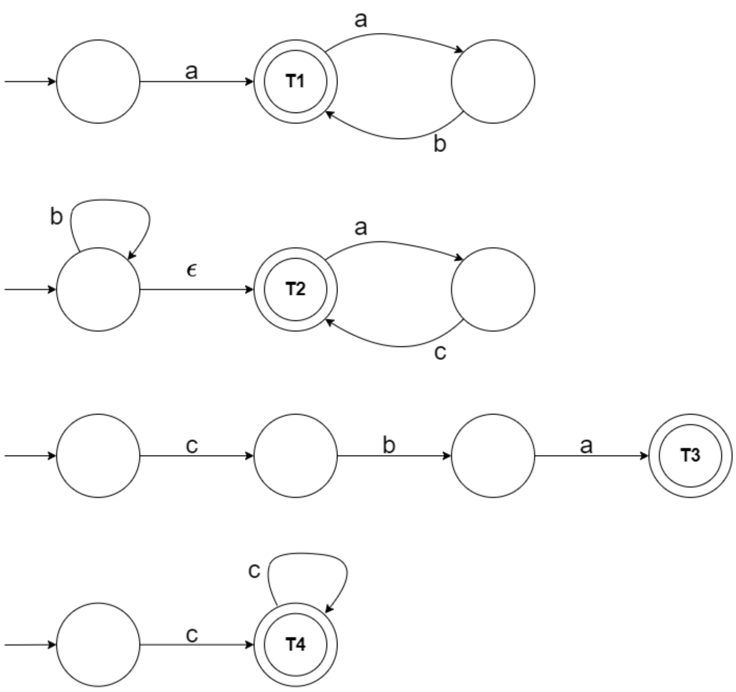
Exercises **3.5** and **3.6** are taken from Basics of Compiler Design, and you can find the solutions here.

Exercise 1

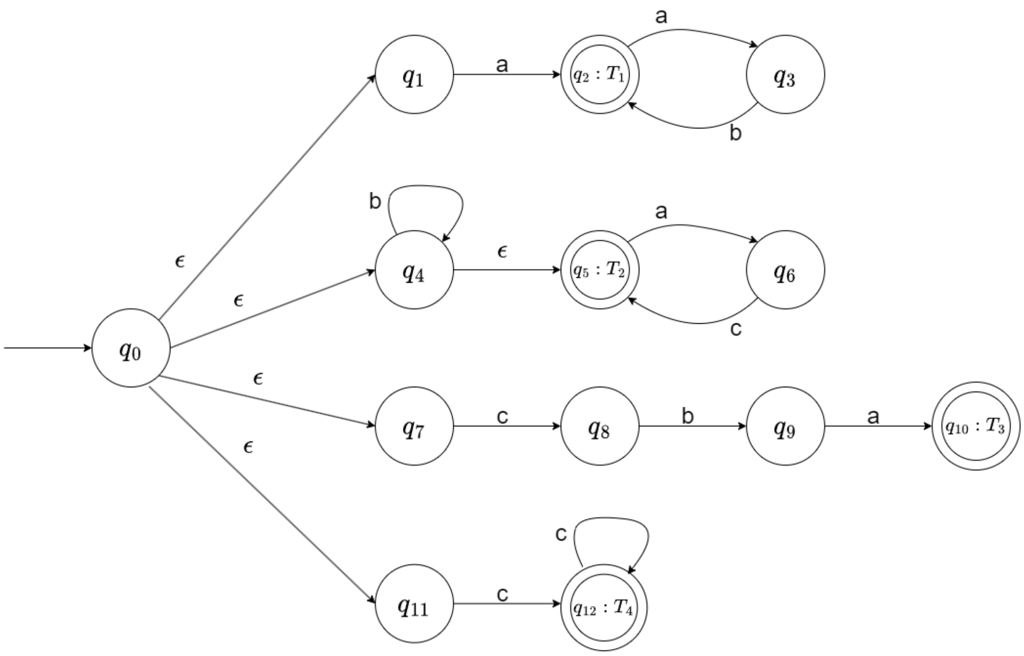
a)
The token sequence of the first word is: T4(c), T2(ac), T4(c), T1(a), T2(bacac), T3(cba), T2(b), T4(c).

For the second word, the token sequence is: T4(ccc), T1(aabab), T2(ac), T3(cba), T2(b), T4(cc), T2(b), T1(a), T2(bac).

b) The corresponding automata are given below.



c) The corresponding NFA is:



Now, let us transform it into a DFA.

The initial state will be $q'_0 = E(q_0) = \{q_0, q_1, q_4, q_5, q_7, q_{11}\}$

From q'_0:
- on $a \to \{q_2, q_6\} := q'_1$
- on $b \to \{q_4, q_5\} := q'_2$
- on $c \to \{q_8, q_{12}\} := q'_3$

From q'_1:
- on $a \to \{q_3\} := q'_4$
- on $b \to \{\} := q'_5$, a trap state
- on $c \to \{q_5\} := q'_6$

From q'_2:
- on $a \to \{q_6\} := q'_7$
- on $b \to \{q_4, q_5\} = q'_2$
- on $c \to \{\} = q'_5$

From q'_3:
- on $a \to \{\} = q'_5$
- on $b \to \{q_9\} := q'_8$
- on $c \to \{q_{12}\} := q'_9$

From q'_4:
- on $a \to \{\} = q'_5$
- on $b \to \{q_2\} := q'_{10}$
- on $c \to \{\} = q'_5$

From q_5':
- on $a \to \{\} = q_5'$
- on $b \to \{\} = q_5'$
- on $c \to \{\} = q_5'$

From q_6':
- on $a \to \{q_6\} = q_7'$
- on $b \to \{\} = q_5'$
- on $c \to \{\} = q_5'$

From q_7':
- on $a \to \{\} = q_5'$
- on $b \to \{\} = q_5'$
- on $c \to \{q_5\} = q_6'$

From q_8':
- on $a \to \{q_{10}\} \coloneqq q_{11}'$
- on $b \to \{\} = q_5'$
- on $c \to \{\} = q_5'$

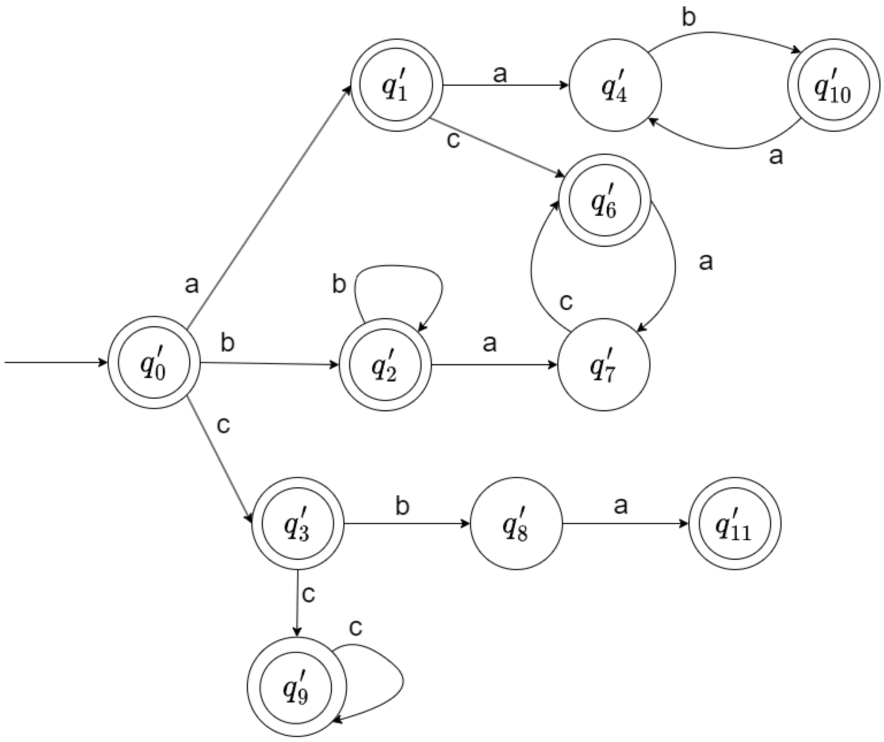
From q_9':
- on $a \to \{\} = q_5'$
- on $b \to \{\} = q_5'$
- on $c \to \{q_{12}\} = q_9'$

From q_{10}':
- on $a \to \{q_3\} = q_4'$
- on $b \to \{\} = q_5'$
- on $c \to \{\} = q_5'$

From q_{11}':
- on $a \to \{\} = q_5'$
- on $b \to \{\} = q_5'$
- on $c \to \{\} = q_5'$

The final states are: $q_0', q_1', q_2', q_3', q_6', q_9', q_{10}', q_{11}'$.

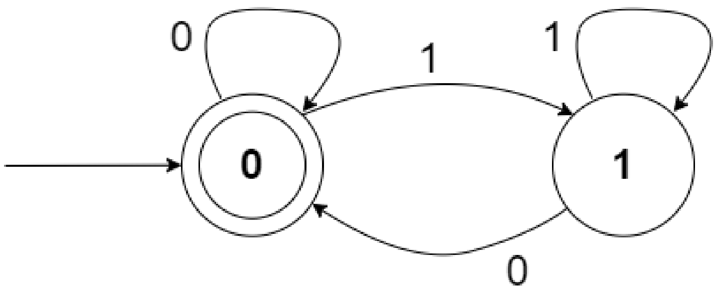
The resulting DFA is[1]:



Exercise 2

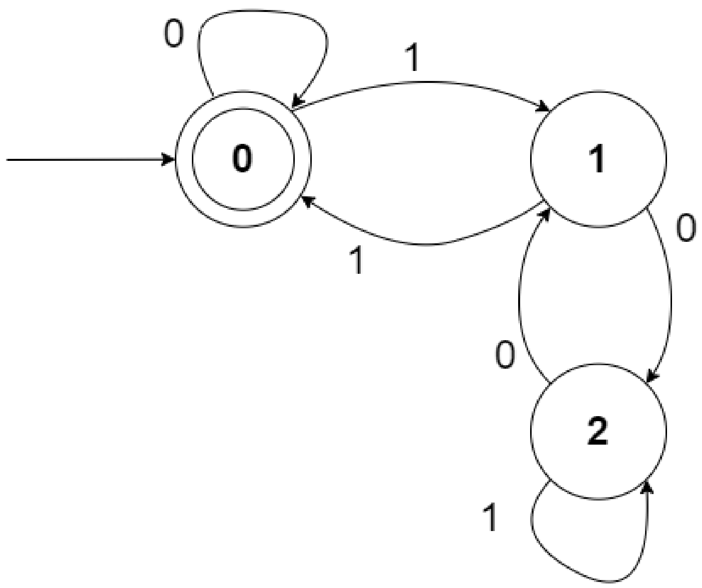
N.B:
- Adding a 0 after a binary number multiplies it by 2
- Adding a 1 after a binary number multiplies it by 2, and then adds 1
- States in the automata will correspond to remainders

a)
The automaton of multiples of 2 is:

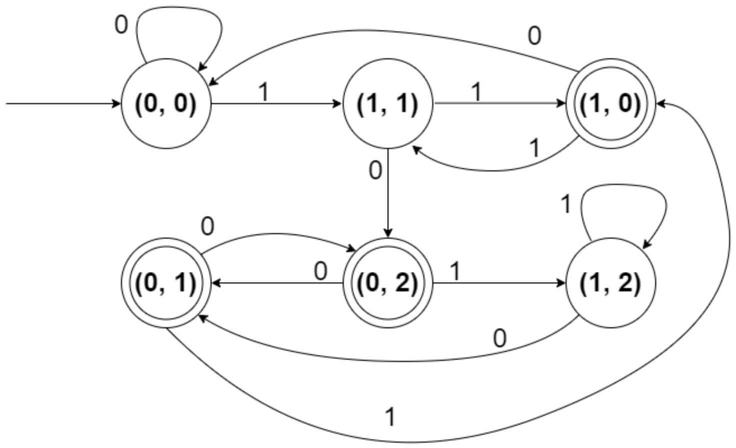


[1]The trap state q_5' is not shown

The automaton of multiples of 3 is:



b), c) The automaton of multiples of either 2 or 3, but not both, is:



The parallel composition has the same DFA, but with the state (0, 0) being final.

Exercise 3

Proof by contradiction.

Assuming L is regular, the pumping lemma applies. Let the word $w \in L$ be of length at least the pumping constant p [2].

According to the lemma, $w = xyz$ with $|y| > 0$. Moreover, for any i, we must have that $xy^iz \in L$, and thus $|xy^iz|$ is prime. We also know that $|xy^iz| = |x| + i|y| + |z| = |xyz| + (i-1)|y|$.

Let us consider the case $i = |xyz| + 1$. We must have:

$|xyz| + |xyz| \cdot |y| = (|y| + 1) \cdot |xyz|$ is prime.

Since $|y| > 0$, both terms of the product are greater than 1, and thus $|xy^iz|$ is not prime, therefore $xy^iz \notin L$ for $i = |xyz| + 1$, which contradicts our initial assumption.

[2] We also pick w with length strictly greater than 1