Computer Language Processing

Exercise Sheet 06 - Solutions

November 7, 2022

Exercise 1

$$\frac{}{\Gamma \vdash \texttt{true} : \texttt{Bool}} \qquad \frac{}{\Gamma \vdash \texttt{false} : \texttt{Bool}} \qquad \frac{\texttt{c} \text{ is a literal integer}}{\Gamma \vdash \texttt{c} : \texttt{Int}}$$

$$\frac{\Gamma \vdash t_1 : \texttt{Int} \quad \Gamma \vdash t_2 : \texttt{Int}}{\Gamma \vdash t_1 \texttt{ == } t_2 : \texttt{Bool}} \qquad \frac{\Gamma \vdash t_1 : \texttt{Bool} \quad \Gamma \vdash t_2 : \texttt{Bool}}{\Gamma \vdash t_1 \texttt{ == } t_2 : \texttt{Bool}}$$

$$\frac{\Gamma \vdash t_1 : \texttt{Int} \quad \Gamma \vdash t_2 : \texttt{Int}}{\Gamma \vdash t_1 \texttt{ + } t_2 : \texttt{Int}} \qquad \frac{\Gamma \vdash t_1 : \texttt{Bool} \quad \Gamma \vdash t_2 : \texttt{Bool}}{\Gamma \vdash t_1 \texttt{ \&\& } t_2 : \texttt{Bool}}$$

$$\frac{\Gamma \vdash t_1 : \texttt{Bool} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \texttt{if (}t_1\texttt{) } t_2 \texttt{ else } t_3 : T}$$

$$\frac{\Gamma(\texttt{x}) = T}{\Gamma \vdash \texttt{x} : T} \qquad \frac{\Gamma \vdash f : (T_1, \ldots, T_n) \texttt{ => } T \quad \Gamma \vdash t_1 : T_1 \quad \ldots \quad \Gamma \vdash t_n : T_n}{\Gamma \vdash f(t_1, \ldots, t_n) : T}$$

Exercise 2

1.

$$\frac{\Gamma \vdash 3 : \textbf{Int} \qquad \Gamma \vdash 5 : \textbf{Int}}{\Gamma \vdash 3 \ + \ 5 : \textbf{Int}}$$

2.

$$\dfrac{\dfrac{}{\emptyset \vdash \texttt{4 : Int}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\{(\texttt{x, Int})\}(\texttt{x}) = \texttt{Int}}{\{(\texttt{x, Int})\} \vdash \texttt{x : Int}} \qquad \dfrac{\{(\texttt{x, Int})\}(\texttt{x}) = \texttt{Int}}{\{(\texttt{x, Int})\} \vdash \texttt{x : Int}}}{\{(\texttt{x, Int})\} \vdash \texttt{x + x : Int}} \qquad \dfrac{\dfrac{\dfrac{\{(\texttt{x, Int}), (\texttt{y, Int})\}(\texttt{x}) = \texttt{Int}}{\{(\texttt{x, Int}), (\texttt{y, Int})\} \vdash \texttt{x : Int}} \qquad \dfrac{\{(\texttt{x, Int}), (\texttt{y, Int})\}(\texttt{y}) = \texttt{Int}}{\{(\texttt{x, Int}), (\texttt{y, Int})\} \vdash \texttt{y : Int}}}{\{(\texttt{x, Int}), (\texttt{y, Int})\} \vdash \texttt{x * y : Int}}}{\{(\texttt{x, Int})\} \vdash \texttt{val y: Int = x + x; x * y : Int}}}{\emptyset \vdash \texttt{val x: Int = 4; val y: Int = x + x; x * y : Int}}$$

3.

$$\Gamma_0 = \{(\texttt{x, Boolean}), (\texttt{power, (Int, Int) => Int})\}$$
$$\Gamma_1 = \{(\texttt{x, Int}), (\texttt{power, (Int, Int) => Int})\}$$

$$\dfrac{\dfrac{\dfrac{\Gamma_0(\texttt{x}) = \texttt{Boolean}}{\Gamma_0 \vdash \texttt{x : Boolean}} \qquad \dfrac{}{\Gamma_0 \vdash \texttt{1 : Int}} \qquad \dfrac{}{\Gamma_0 \vdash \texttt{0 : Int}}}{\Gamma_0 \vdash \texttt{if (x) 1 else 0 : Int}} \qquad \dfrac{\dfrac{\Gamma_1(\texttt{x}) = \texttt{Int}}{\Gamma_1 \vdash \texttt{x : Int}} \qquad \dfrac{}{\Gamma_1 \vdash \texttt{2 : Int}}}{\Gamma_1 \vdash \texttt{x * 2 : Int}}}{\Gamma_0 \vdash \texttt{val x: Int = if (x) 1 else 0; x * 2 : Int}}$$

4.

$$\Gamma_0 = \{(\texttt{x, Boolean}), (\texttt{power, (Int, Int) => Int})\}$$
$$\Gamma_1 = \{(\texttt{x, Int}), (\texttt{power, (Int, Int) => Int})\}$$

$$\dfrac{\dfrac{}{\Gamma_0 \vdash \texttt{7 : Int}} \qquad \dfrac{\dfrac{\dfrac{\Gamma_1(\texttt{x}) = \texttt{Int}}{\Gamma_1 \vdash \texttt{x : Int}} \qquad \dfrac{}{\Gamma_1 \vdash \texttt{100 : Int}}}{\Gamma_1 \vdash \texttt{x < 100 : Boolean}} \qquad \dfrac{\vdots}{\Gamma_1 \vdash \texttt{power(x, 10) : Int}} \qquad \dfrac{}{\Gamma_1 \vdash \texttt{error("Too big!") : Int}}}{\Gamma_1 \vdash \texttt{if (x < 100) power(x, 10) else error("Too big!") : Int}}}{\Gamma_0 \vdash \texttt{val x: Int = 7; if (x < 100) power(x, 10) else error("Too big!") : Int}}$$

Where, for space reasons...

$$\dfrac{\dfrac{\Gamma_1(\texttt{f}) = \texttt{(Int, Int) => Int}}{\Gamma_1 \vdash \texttt{f : (Int, Int) => Int}} \qquad \dfrac{\Gamma_1(\texttt{x}) = \texttt{Int}}{\Gamma_1 \vdash \texttt{x : Int}} \qquad \dfrac{}{\Gamma_1 \vdash \texttt{10 : Int}}}{\Gamma_1 \vdash \texttt{power(x, 10) : Int}}$$

Exercise 3

Valid answers:

C. **There does not exist valid derivations where T_1 is `Int`.**

D. **In all valid derivations, T_2 is equal to (T_4, T_5).**

Exercise 4

Infer the type of the following expressions:

1. `Int => Int`
2. `Int => Int => Int`
3. `Int => (Int => Int)`
4. `Int => Int`
5. `No Type`
6. `Bool => (Int => Int)`
7. `((Int => Bool) => Bool) => (Int => Bool) => Bool`
8. `((Int => Int) => Int) => (Int => Int) => Int => Int`