Computer Language Processing

Exercise Sheet 04 - Solutions

October 26, 2022

Exercise 1

The minimal subset is the following: $\{B, E, H, K, L\}$

Exercise 2

1)

$$\frac{}{\texttt{true}[x := e] \to \texttt{true}} \qquad \frac{}{\texttt{false}[x := e] \to \texttt{false}} \qquad \frac{\texttt{c} \text{ is a literal integer}}{\texttt{c}[x := e] \to \texttt{c}}$$

$$\frac{t_1[x := e] \to t_1' \qquad t_2[x := e] \to t_2'}{(t_1 \texttt{ == } t_2)[x := e] \to (t_1' \texttt{ == } t_2')}$$

$$\frac{t_1[x := e] \to t_1' \qquad t_2[x := e] \to t_2'}{(t_1 \texttt{ + } t_2)[x := e] \to (t_1' \texttt{ + } t_2')} \qquad \frac{t_1[x := e] \to t_1' \qquad t_2[x := e] \to t_2'}{(t_1 \texttt{ \&\& } t_2)[x := e] \to (t_1' \texttt{ \&\& } t_2')}$$

$$\frac{t_1[x := e] \to t_1' \qquad t_2[x := e] \to t_3' \qquad t_1[x := e] \to t_3'}{(\texttt{if } (t_1)\ t_2 \texttt{ else } t_3)[x := e] \to (\texttt{if } (t_1')\ t_2' \texttt{ else } t_3')}$$

$$\frac{x = y}{\texttt{y}[x := e] \to e} \qquad \frac{x \neq y}{\texttt{y}[x := e] \to \texttt{y}} \qquad \frac{t_1[x := e] \to t_1' \qquad \dots \qquad t_n[x := e] \to t_n'}{f(t_1, \dots, t_n)[x := e] \to f(t_1', \dots, t_n')}$$

2)

Function calls

$$\frac{\begin{array}{c} b_0 \text{ is the body of } f \text{ and } x_i \text{ are the } n \text{ parameters of } f \\ b_0[x_1 := t_1] \to b_1 \qquad \dots \qquad b_{n-1}[x_n := t_n] \to b_n \end{array}}{f(t_1, \dots, t_n) \rightsquigarrow b_n}$$

&& **operator**

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 \ \&\& \ t_2 \rightsquigarrow t_1' \ \&\& \ t_2}$$

$$\frac{b \text{ is a boolean and } t_2 \rightsquigarrow t_2'}{b \ \&\& \ t_2 \rightsquigarrow b \ \&\& \ t_2'}$$

$$\frac{b \text{ is a boolean}}{true \ \&\& \ b \rightsquigarrow b}$$

$$\frac{b \text{ is a boolean}}{false \ \&\& \ b \rightsquigarrow false}$$

+ operator

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 \ + \ t_2 \rightsquigarrow t_1' \ + \ t_2}$$

$$\frac{n \text{ is an integer and } t_2 \rightsquigarrow t_2'}{n \ + \ t_2 \rightsquigarrow n \ + \ t_2'}$$

$$\frac{n_1, n_2 \text{ are integers} \qquad n_1 + n_2 = n_3}{n_1 + n_2 \rightsquigarrow n_3}$$

== operator

$$\frac{t_1 \rightsquigarrow t_1'}{t_1 \ == \ t_2 \rightsquigarrow t_1' \ == \ t_2}$$

$$\frac{n \text{ is an integer and } t_2 \rightsquigarrow t_2'}{n \ == \ t_2 \rightsquigarrow n \ == \ t_2'}$$

$$\frac{n_1, n_2 \text{ are integers} \qquad n_1 = n_2}{n_1 == n_2 \rightsquigarrow true}$$

$$\frac{n_1, n_2 \text{ are integers} \qquad n_1 \neq n_2}{n_1 == n_2 \rightsquigarrow false}$$

If/else

$$\frac{t_1 \rightsquigarrow t_1'}{if \ (t_1) \ t_2 \ else \ t_3 \rightsquigarrow if \ (t_1') \ t_2 \ else \ t_3}$$

$$\frac{}{if\ (true)\ t_2\ else\ t_3 \rightsquigarrow t_2}$$

$$\frac{}{if\ (false)\ t_2\ else\ t_3 \rightsquigarrow t_3}$$

Exercise 3

1) At each step we apply one of the rules. Note that we do not reduce the terms inside the body of a lambda.

$$\begin{array}{l}
(\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, ((\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\; z)) \\
(\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z)) \\
\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z))\; s\; z)
\end{array}$$

2) We only need two rules (in total) for the new semantics:

$$\frac{t_1 \rightsquigarrow t_1'}{t_1\; t_2 \rightsquigarrow t_1'\; t_2} \qquad \text{(App1)}$$

$$(\lambda x.\, t_1)\; t_2 \rightsquigarrow t_1[x \mapsto t_2] \qquad \text{(AppAbsByName)}$$

We can now evaluate our expression:

$$\begin{array}{l}
(\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, ((\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\; z)) \\
\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z)
\end{array}$$

3) We need a rule that allows us to reduce the body of a lambda. We add the following rule:

$$\frac{t_1 \rightsquigarrow t_1'}{\lambda x.\, t_1 \rightsquigarrow \lambda x.\, t_1'} \qquad \text{(AbsBody)}$$

We now obtain, as expected:

$$\begin{array}{l}
(\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, ((\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\; z)) \\
\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, n.\, \lambda\, s.\, \lambda\, z.\, s\, (n\; s\; z))\, (\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z) \\
\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z))\; s\; z) \\
\lambda\, s.\, \lambda\, z.\, s\, ((\lambda\, z.\, s\, ((\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z))\; z) \\
\lambda\, s.\, \lambda\, z.\, s\, (s\, ((\lambda\, s.\, \lambda\, z.\, s\; z)\; s\; z)) \\
\lambda\, s.\, \lambda\, z.\, s\, (s\, ((\lambda\, z.\, s\; z)\; z)) \\
\lambda\, s.\, \lambda\, z.\, s\, (s\, (s\; z))
\end{array}$$