

# Computer Language Processing

## Exercise Sheet 06 - Solutions

November 4, 2022

### Exercise 1

$$\begin{array}{c} \frac{}{\Gamma \vdash \text{true} : \text{Bool}} \quad \frac{}{\Gamma \vdash \text{false} : \text{Bool}} \quad \frac{\text{c is a literal integer}}{\Gamma \vdash \text{c} : \text{Int}} \\[10pt] \frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 == t_2 : \text{Bool}} \quad \frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : \text{Bool}}{\Gamma \vdash t_1 == t_2 : \text{Bool}} \\[10pt] \frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 + t_2 : \text{Int}} \quad \frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : \text{Bool}}{\Gamma \vdash t_1 \&\& t_2 : \text{Bool}} \\[10pt] \frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \text{if } (t_1) \ t_2 \ \text{else } t_3 : T} \\[10pt] \frac{\Gamma(\mathbf{x}) = T}{\Gamma \vdash \mathbf{x} : T} \quad \frac{\Gamma \vdash f : (T_1, \dots, T_n) \Rightarrow T \quad \Gamma \vdash t_1 : T_1 \quad \dots \quad \Gamma \vdash t_n : T_n}{\Gamma \vdash f(t_1, \dots, t_n) : T} \end{array}$$

## Exercise 2

1.

$$\frac{\Gamma \vdash 3 : \text{Int} \quad \Gamma \vdash 5 : \text{Int}}{\Gamma \vdash 3 + 5 : \text{Int}}$$

2.

$$\frac{\frac{\frac{\frac{\{(x, \text{Int})\}(x) = \text{Int}}{\{(x, \text{Int})\} \vdash x : \text{Int}} \quad \frac{\{(x, \text{Int})\}(x) = \text{Int}}{\{(x, \text{Int})\} \vdash x : \text{Int}}}{\{(x, \text{Int})\} \vdash x + x : \text{Int}} \quad \frac{\frac{\frac{\{(x, \text{Int}), (y, \text{Int})\}(x) = \text{Int}}{\{(x, \text{Int}), (y, \text{Int})\} \vdash x : \text{Int}} \quad \frac{\{(x, \text{Int}), (y, \text{Int})\}(y) = \text{Int}}{\{(x, \text{Int}), (y, \text{Int})\} \vdash y : \text{Int}}}{\{(x, \text{Int}), (y, \text{Int})\} \vdash x * y : \text{Int}}}{\frac{\emptyset \vdash 4 : \text{Int} \quad \{(x, \text{Int})\} \vdash \text{val } y : \text{Int} = x + x; x * y : \text{Int}}{\emptyset \vdash \text{val } x : \text{Int} = 4; \text{val } y : \text{Int} = x + x; x * y : \text{Int}}}$$

3.

$$\frac{\Gamma_0 = \{(x, \text{Boolean}), (\text{power}, (\text{Int}, \text{Int}) \Rightarrow \text{Int})\} \quad \Gamma_1 = \{(x, \text{Int}), (\text{power}, (\text{Int}, \text{Int}) \Rightarrow \text{Int})\}}{\frac{\frac{\frac{\Gamma_0(x) = \text{Boolean}}{\Gamma_0 \vdash x : \text{Boolean}} \quad \frac{\Gamma_0 \vdash 1 : \text{Int}}{\Gamma_0 \vdash 1 : \text{Int}} \quad \frac{\Gamma_0 \vdash 0 : \text{Int}}{\Gamma_0 \vdash 0 : \text{Int}}}{\Gamma_0 \vdash \text{if } (x) \text{ } 1 \text{ else } 0 : \text{Int}} \quad \frac{\frac{\frac{\Gamma_1(x) = \text{Int}}{\Gamma_1 \vdash x : \text{Int}} \quad \frac{\Gamma_1 \vdash 2 : \text{Int}}{\Gamma_1 \vdash 2 : \text{Int}}}{\Gamma_1 \vdash x * 2 : \text{Int}}}{\Gamma_0 \vdash \text{val } x : \text{Int} = \text{if } (x) \text{ } 1 \text{ else } 0; x * 2 : \text{Int}}}$$

4.

$$\frac{\Gamma_0 = \{(x, \text{Boolean}), (\text{power}, (\text{Int}, \text{Int}) \Rightarrow \text{Int})\} \quad \Gamma_1 = \{(x, \text{Int}), (\text{power}, (\text{Int}, \text{Int}) \Rightarrow \text{Int})\}}{\frac{\frac{\frac{\frac{\Gamma_1(x) = \text{Int}}{\Gamma_1 \vdash x : \text{Int}} \quad \frac{\Gamma_1 \vdash 100 : \text{Int}}{\Gamma_1 \vdash 100 : \text{Int}} \quad \vdots}{\Gamma_1 \vdash x < 100 : \text{Boolean}} \quad \frac{\Gamma_1 \vdash \text{power}(x, 10) : \text{Int}}{\Gamma_1 \vdash \text{power}(x, 10) : \text{Int}} \quad \frac{\Gamma_1 \vdash \text{error}(\text{"Too big!"}) : \text{Int}}{\Gamma_1 \vdash \text{error}(\text{"Too big!"}) : \text{Int}}}{\frac{\Gamma_0 \vdash 7 : \text{Int} \quad \Gamma_1 \vdash \text{if } (x < 100) \text{ power}(x, 10) \text{ else error}(\text{"Too big!"}) : \text{Int}}{\Gamma_0 \vdash \text{val } x : \text{Int} = 7; \text{if } (x < 100) \text{ power}(x, 10) \text{ else error}(\text{"Too big!"}) : \text{Int}}}$$

Where, for space reasons...

$$\frac{\frac{\Gamma_1(f) = (\text{Int}, \text{Int}) \Rightarrow \text{Int}}{\Gamma_1 \vdash f : (\text{Int}, \text{Int}) \Rightarrow \text{Int}} \quad \frac{\Gamma_1(x) = \text{Int}}{\Gamma_1 \vdash x : \text{Int}} \quad \frac{\Gamma_1 \vdash 10 : \text{Int}}{\Gamma_1 \vdash 10 : \text{Int}}}{\Gamma_1 \vdash \text{power}(x, 10) : \text{Int}}$$

## Exercise 3

Valid answers:

- C. **There does not exist valid derivations where  $T_1$  is Int.**
- D. **In all valid derivations,  $T_2$  is equal to  $(T_4, T_5)$ .**

## Exercise 4

Infer the type of the following expressions:

1. `Int => Int`
2. `Int => Int => Int`
3. `Int => (Int => Int)`
4. `Int => Int`
5. `No Type`
6. `Bool => (Int => Int)`
7. `((Int => Bool) => Bool) => (Int => Bool) => Int`
8. `((Int => Int) => Int) => (Int => Int) => Int => Int`