
Functional Programming

Final Exam Solution

Friday, December 21 2018

Exercise 1: Pure Functional Programming (10 points)

Non tail recursive solution (7 points)

```
def flatMap[T](list: List[T], f: T => List[T]): List[T] = {
  list match {
    case x :: xs => f(x) ::: flatMap(xs, f)
    case Nil => Nil
  }
}
```

Tail recursive solution (10 points)

```
def flatMap[T](list: List[T], f: T => List[T]): List[T] = {
  @tailrec def reverse(ls: List[T], acc: List[T]): List[T] = {
    ls match {
      case x :: xs => reverse(xs, x :: acc)
      case Nil => acc
    }
  }
  @tailrec def rec(ls: List[T], acc: List[T]): List[T] = {
    ls match {
      case x :: xs => rec(xs, f(x) ::: acc)
      case Nil => acc
    }
  }
  rec(reverse(list, Nil), Nil)
}
```

Exercise 2: State (10 points)

1. $f1(n)$: **Y**
all operations used are RT (referentially transparent)
2. $f2(n, m)$: **Y**
all operations used are RT
3. $f3(xs, _ + _)$: **Y**
all operations used are RT and the mutable variables are local
4. $f3(xs, _ + c.get + _)$: **N**
 $c.get$ is not RT
5. $f4()$: **Y**
all operations used are RT
6. $f5()$: **N**
 $println$ is not RT
7. $f6(c)$: **N**
 $c.inc$ is not RT
8. $f6(new Counter)$: **N**
the returned Counter holds state
9. $f6(new Counter).get$: **Y**
the Counter is local to the expression (its state does not leak)
10. $f7(c)$: **N**
 $c.get$ is not RT
11. $f8(n)(c)$: **N**
 $c.inc$ is not RT
12. $f8(n)$: **Y**
the function is not fully applied and the partial application has no state
13. $f8(c.get)$: **N**
 $c.get$ is not RT
14. $f9((x:Int) => (), c.get)$: **Y**
while $c.get$ is not RT, its result is discarded and does not influence the program
15. $f9(f1, f1(c.get))$: **N**
 $c.get$ is not RT
16. $f9(x => y => println(x+y), 0)$: **Y**
the function is not fully applied and the partial application has no state
17. $f10(f1)$: **Y**
all operations used are RT and the local cache is not observable
18. $f10(x => c.inc.get + x)$: **N**
 $c.int$ and $c.get$ are not RT
19. $f10(x => c.get + x)$: **N**
 $c.get$ is not RT
20. $f11$: **Y**
the local counter's state is never changed, so the function passed to $f10$ in $f11$ is RT, and $f11$ is RT

Exercise 3: Lambda Calculus (10 points)

3.1

```
def (succ n)
  (lambda (f x) (f (n f x)))
```

3.2

```
def (size list)
  (list
   zero
   (lambda (h t) (succ (size t))))
)
```

Exercise 4: Streams (10 points)

```
def trans(src: Stream[Stream[String]], base: Int, n: Int): Stream[String] =
  src.drop(n - 1).head.drop(base - 1).head #:: trans(src, base, n + 1)
```

```
def transposed(src: Stream[Stream[String]], x: Int, y: Int): Stream[Stream[String]] =
  trans(src, x, y) #:: transposed(src, x + 1, y)
```

```
def transpose(src: Stream[Stream[String]]): Stream[Stream[String]] =
  transposed(src, 1, 1)
```